FIBONACCI NUMBERS FOR PALM FOLIAR SPIRALS

T. A. DAVIS

Indian Statistical Institute, Calcutta, India

SUMMARY

The Fibonacci sequence (1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144 . . .) formulated by the Italian mathematician, Leonardo da Pisa has many interesting mathematical properties. One of these is the fairly constant proportion that works out between any two consecutive stages in the sequence excluding, of course, some lower stages. Such a proportion, spoken of as Golden Proportion, exists in many plant species having alternate phyllotaxis. The foliar arrangement in a few species of palms has been critically studied with a view to understanding the mathematics underlying it.

Palms display one, two, three, five, eight, or thirteen foliar spirals, and the spirals in a palm may move clockwisely or counter-clockwisely, each species having both types of palms in almost equal proportions. *Areca catechu, Arenga pinnata, Borassus flabellifer, Cocos nucifera, Elaeis guineensis, Phoenix canariensis* respectively are examples for the above foliar spiral categories. All these numbers synchronise the Fibonacci Numbers, and no palm is known to have foliar spirals numbering 4, 6, 7, 9, 10, 11 or 12. Exceptional species, however, do not show any clear spirals, but their leaves are arranged vertically in two, three, five or eight rows, the numbers again synchronising the Fibonacci stages.

A model has been suggested for the arrangement of leaves in any palm crown in which two consecutive leaves have been shown to subtend 137.5° between them, and so this angle makes a proportion of 0.618 with the remaining angle (222.5°) to complete one full revolution. It is this Golden Proportion that is responsible for palm leaves appearing in spirals.

An explanation is offered for the apparent reversed situations in the positions of the flower-bunches in the coconuts (in left- and right-spiralled palms) and in the African oil palm.

The drawing was prepared by Mr. S. K. De, the Senior Technical Assistant of the above Institute.

1. INTRODUCTION

Palms are ideal material to study the phyllotaxis since the very large, prominently-petiolate leaves are arranged compactly into a crown at the tip of the usually unbranched trunk which, in addition, bears leaf bases and/or legible scars of the leaves already shed. The arrangement of palm leaves is distinctly alternate, falling in line with the majority of plants that possess spiral phyllotaxis and conforming to the Schimper-Braun series of divergences such as 2/5, 3/8, 5/13, 8/21, 13/34, 21/55 . . . . or 2/3, 3/5, 5/8, 8/13, 13/21, 21/34, 34/55, and so on. The numerators or the denominators of this series, when considered alone, form the successive stages of a numerical sequence, formulated by the thirteenth century Italian mathematician, Leonardo da Pisa, known popularly as Fibonacci Numbers, which run as: 1,1, 2,3, 5,8, 13, 21, 34, 55, 89, 144, and so on. It is known (CHURCH 1904, THOMPSON 1917, RICHARDS 1951, LEPPIK 1961) that the Fibonacci phyllotaxis gives optimum illumination to the photosynthetic surface of plants.

2. VARYING NUMBERS OF FOLIAR SPIRALS IN PALMS

Different species of palms show different numbers of foliar spirals, and the numbers always match with the Fibonacci Numbers. For example, in *Areca catechu* (*fig. 1*) or *Ptychosperma macarthurii*, only a single foliar spiral is discernible while *Arenga pinnata* or *A. saccharifera* (*fig. 2*) has two spirals. *Borassus flabellifer* (*fig. 3*), *Corypha elata* and a number of other species of palms have three spirals each. *Cocos nucifera* (*fig. 4*) has five spirals, while *Elaeis guineensis* (*fig. 5*) bears eight spirals. *Phoenix canariensis* (*fig. 6*) has thirteen spirals. If the huge trunks of some *P. canariensis* are examined critically, it is sometimes possible to trace out the twentyone spirals running obliquely across the thirteen spirals. The size of the trunks of palms being limited and their leaves being usually very large, it is difficult to find examples of palms having leaf spirals matching with still higher numbers of the Fibonacci sequence. It is known that large heads of *Helianthus annuus* may show 34, 55, 89 or even 144 spirals in the arrangement of the disc florets.

Palms bearing 4, 6, 7, 9, 10, 11 or 12 obvious foliar spirals are not known. A hypothetical explanation for the number of foliar spirals in a palm always synchronising with a stage in the Fibonacci sequence is offered in this report.

The number of green leaves a palm bears at a time generally indicates the number of foliar spirals exhibited by the species. Palms having fewer leaves manifest smaller number of foliar spirals, and those bearing larger numbers of leaves show greater number of spirals. This situation can be easily explained by the help of the schematic representation of a palm crown shown in *fig. 7*.

3. SCHEMATIC REPRESENTATION OF A PALM CROWN

The centralmost point in the drawing (*fig. 7*) represents the aerial view of a palm trunk, and the radial lines, its leaves. The outermost leaf which is the oldest, is numbered 1. Leaf No 2 is drawn at an arbitrary angular deflection of 137.5° to the left of leaf No. 1. Since leaf No. 2 is nearer to leaf No. 1 by the left-hand side of an observer looking from the tip of leaf No. 1, this crown may be regarded as representing a left-spiralled palm. Similarly, leaf No. 3 will be nearer to leaf No. 2 by the left, and the subsequent leaves are also similarly located. In another palm, leaf No. 2 can as well be nearer to leaf No. 1 by the right, in which case the diagram will represent a right-spiralled crown. The two types of individuals for any species of palms are distributed more or less equally in any locality although for *Cocos nucifera* there is an excess of one kind of individuals in the Northern hemisphere, while the Southern hemisphere has more of the other kind, the hemispherical differences being significant statistically (DAVIS 1964).

The younger leaves are represented progressively by shorter radial lines, and leaf No. 48 is the youngest visible leaf in this crown. The tips of all the leaves are connected by a line which forms a clockwise (left-handed) coil, and this will represent the only visible spiral in some palms such as *Areca catechu*.

Plate I fig. 1. A left-spiralled *Areca catechu*; fig. 2. Two-spiralled trunk of *Arenga saccharifera*; fig. 3. Stem of *Borassus flabellifer* having three spirals; fig. 4. The crown of *Cocos nucifera* showing left-handed foliar spiral; fig. 5. Trunk of *Elaeis guineensis* displaying eight and five spirals.

In a palm showing two foliar spirals, one spiral will comprise leaves 1, 3, 5, 7, 9, and so on, while the second spiral will comprise all the even-number leaves. It is to be noted that both the spirals move counterclockwisely. In a palm bearing three clearly visible spirals, the following leaves constitute the three spirals. Spiral one will connect leaves 1, 4, 7, 10, 13, and so on. The second spiral will have leaves 2, 5, 8, 11, 14, and so on, while the third spiral will comprise leaves 3, 6, 9, 12, 15, and so on. All the three spirals run clockwisely as opposed to the direction of the two spirals. No palm is showing four clear spirals. This is in part due to the fact that the leaves 1 and 5 which should form the two consecutive leaves of one of the four spirals are located almost opposite to each other. In a five-spiralled crown, leaves 1, 6, 11, 16, 21, and so on, constitute one of the spirals, the other four starting with leaves 2, 3, 4 and 5 respectively. All the five spirals clearly move counter-clockwisely which is opposite to the direction of the three spirals. In a palm with eight spirals, leaves 1, 9, 17, 25, 33, and so on, will form one of the spirals and the remaining spirals commence from leaves 2, 3, 4, 5, 6, 7 and 8. The eight spirals move opposite to the five spirals. In a like manner, if the diagram is to represent a thirteen spiralled palm (one spiral comprising leaves 1, 14, 27, 40, etc.), the spirals will move opposite to the eight spirals, and so, the twenty one spirals move slantingly opposite to the thirteen spirals. Thus, in this diagram, the more obvious numbers of foliar spirals synchronise with the Fibonacci Numbers. Foliar spirals representing the Fibonacci Numbers 1, 3, 8, 21 move clockwisely. This situation is in conformity with some specific mathematical properties of the Fibonacci sequence.

In a palm crown having four or five leaves, only the single spiral is discernible and two spirals may be clear if the leaf number goes up to seven or eight. Three spirals may be made out in a crown having about ten to twelve leaves, and five spirals in a crown having about twenty leaves. Therefore, as the number of green leaves in a crown increases (a condition which normally necessitates a proportional increase in the girth of the trunk), higher orders of foliar spirals are displayed. Moreover, from a crown showing, say, eight spirals, those representing the two neighbouring F. Numbers (5 and 13) could also be made out. The leaf bases on the *Elaeis guineensis* trunk (*fig. 5*) and the leaf scars on *Phoenix canariensis* (*fig. 6*) stem bear testimony to this.

4. THE GOLDEN PROPORTION

The angular deflection of 137.5° has been chosen arbitrarily because this would provide no two leaves in the diagram exactly superimposing each other till the 145th leaf. No palm is likely to have one hundred functioning leaves at a time, and so, this angular deflection gives the leaves scope for maximum exposure to sunlight. Contrary to the above theoretical concept, there are exceptional species which do not show any spiral arrangement of leaves even though they are alternate. Instead, they are arranged in vertical rows, the number of rows in a species remaining a Fibonacci Number. In *Wallichia disticha* (*fig. 8*), there are only two opposite rows of leaves as in *Ravenala madagasca-*

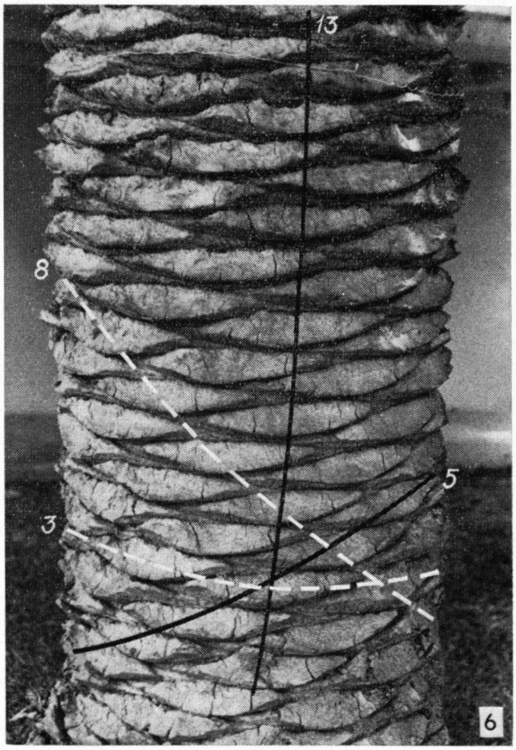

Plate II fig. 6. The leaf scars on the trunk of *Phoenix canariensis* manifest three, five, eight and thirteen foliar spirals.

riensis or more typically as in most grasses. Here the angular deflection between two consecutive leaves is 180°. In *Neodypsis decaryi* (*fig. 9*), the leaves fall in three vertical rows, two successive leaves subtending an angle of 120°. In some individual *Syagrus treubiana* stems (*fig. 10*), five impressive vertical rows of persistent leaf bases which include the green leaves at the apex may be seen, and the narrow angular deflection between two successive leaves will be 144°. If thousands of trunks of *Phoenix sylvestris* or *Elaeis guineensis* which normally show eight spirals are examined, it is possible to come across exceptional individuals showing eight vertical rows at least upto certain lengths of the trunks. In such situations, two consecutive leaves will be deflected at an angle of 135°. Similarly, it is possible to get examples for palm trunks showing thirteen and twentyone vertical rows, and each leaf in them will be disposed at 138.5° and 137.14° respectively from its immediate younger one. If the figures of the above mentioned angular deflections (180°, 120°, 144°, 135°, 138.5°, 137.14° . . .) are examined, one finds the alternate numbers turning out to be more than 137.5°

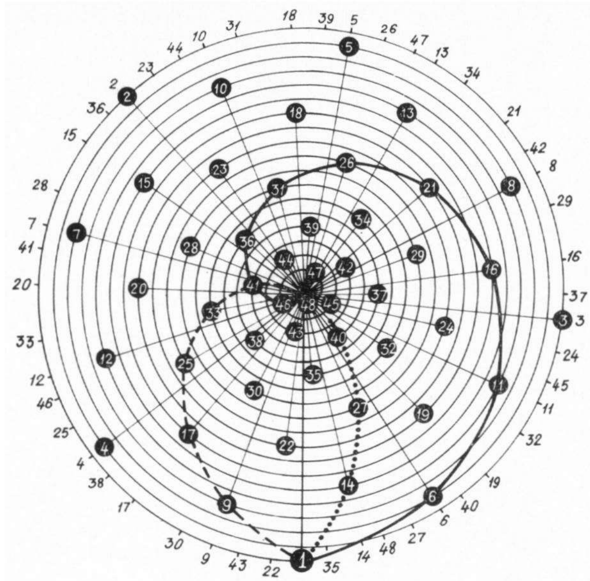

Plate III fig. 7. A schematic representation of the crown of a palm.

and the others less, and the difference between two numbers progressively gets narrower, ultimately reaching 137.5°. This narrow angle (137.5°) makes a proportion of 0.618 with the remaining wider angle (222.5°) to complete one full revolution. This is the familiar Golden Proportion (or Divine Proportion) observable in many biological situations but whose occurrence is inexplicable. A property of the Fibonacci sequence is that the proportion between any two consecutive numbers (excepting the few lower stages) turns out to be Golden Proportion.

Though devoid of secondary thickening (being a monocot), the lowermost part of the trunk of many palms is thicker than the rest, and its internodes much narrower. Therefore, the spirals on the thicker part of the stem will be relatively less steep than those at the upper thinner part of the stem. Thus, variation in the alignment of foliar spirals can be noticed between parts of the same trunk, between trunks of different individuals of the same species and between different species. However, a great majority of individuals of each species display more or less the same kind and similar pattern of foliar spirals. The spirals of palms representing smaller numbers of foliar spirals are relatively flat (showing greater torsion), and those of palms having higher numbers of spirals move more and more steeply. As a consequence, the leaves of palms with smaller numbers of foliar spirals are more asymmetric bilaterally (MITRA 1968).

Plate IV fig. 8. *Wallichia disticha* with two vertical rows of leaves; fig. 9. *Neodypsis decaryi* showing three vertical rows of leaves; fig. 10. Trunk of *Syagrus treubiana* bearing five vertical rows of leaf scars.

In palms like *Nypa fruiticans* which do not form an aerial trunk, the leaves that are produced from the horizontally spreading rhizomes, grow slantingly upwards one alternating with another in a zig-zag manner. Any two successive leaves subtend a narrow angle (upwards) that bears more or less a Golden Proportion to the remaining wider angle.

5. POSITION OF SPADIX AND FOLIAR SPIRAL

In a bearing palm, almost every leaf subtends a spadix which may be female, male or androgynous. This bunch is always younger than the leaf from whose axil it develops. The tendency of the spadix is to lean towards the direction of the leaf just younger to its own subtending leaf. That is, in *fig. 7*, the bunch of leaf No. 1 will point its tip towards the direction of leaf No. 2. The bunch may hang downwards as in the case of *Cocos nucifera* or may point upwards at various angles as in *Elaeis guineensis.* If this figure is regarded as the crown of a coconut representing the five spirals running right-handedly, the bunch will appear moving away from the direction of the leaf spirals. Thus, in a right-spiralled coconut, as in *fig. 7*, the spadix falls on the left side of the subtending leaf (as it appears to an observer looking from below a leaf and towards its stalk), and the spadix hangs to the right of the leaf in a left-spiralled palm (DAVIS 1961). On the other hand, if the diagram is to represent the crown of *Elaeis guineensis* with its eight spirals running clockwisely, the bunch and the foliar spirals will seem to move along the same direction. Thus, the situations in *Cocos* and *Elaeis* are reversed as they represent foliar spirals corresponding to two consecutive stages of the Fibonacci sequence. Since the coconut and the oil palm are grown together in many tropical countries, often confusion arises with regard to the determination of the foliar spirals in these species.

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