

## A MODEL FOR PLANT GROWTH. ESTIMATION OF THE PARAMETERS OF THE LOGISTIC FUNCTION

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### SUMMARY

The logistic function is an adequate model for the growth of individuals of higher plants under constant environmental conditions. The parameters are estimated by means of a simple regression analysis. As an illustration growth of two related annual species (*Galinsoga ciliata* and *G. parviflora*) under heated green-house conditions has been compared.

### 1. INTRODUCTION

In plant ecology and agricultural science growth behaviour of plants is frequently compared. These can be either plants of a different genetical constitution growing in an identical environment, or plants belonging to the same species/ecotype but grown under different conditions. In all such cases an estimate to quantify the integral growth curve may be needed.

Whenever a mathematical formula is applied, the underlying model is to be: (1) formulated explicitly, (2) considered and finally (3) accepted. The present paper presents a growth function with its underlying model, having considerable importance in econometry (TINTNER 1960) and ecology of animal populations (REDDINGIUS 1971). Although KENDALL (1949) stated justly that handling of stochastic logistic processes leads to mathematical difficulties and testing of hypotheses is difficult or impossible (REDDINGIUS 1971), it seems worth investigating if the function gives a useful tool for quantifying behaviour of higher plants or plant parts under stable environmental conditions.

### 2. METHOD

The current model for over-all analysis of plant growth is the Mean Relative Growth Rate (R. G. R.) (WILLIAMS 1946; WHITEHEAD & MYERSCOUGH 1962; VAN DOBBEN 1967; EVANS 1972). The only presupposition is:

The rate of growth in any time interval is directly proportional to the actual weight.

As a differential equation:

$$\frac{dY}{dt} = rY, \quad (r > 0) \quad (1)$$

Integrated formula:

$$Y_t = e^{(rt + s)} \quad (2)$$

or

$$\ln Y_t = rt + s \quad (3)$$

This curve is logarithmic and not asymptotic. Since a plant growth curve is in general sigmoid and asymptotic, the R. G. R. can only be used as an approximation for the first part of it. For analysing growth in relation to final weight, R.G.R. cannot be used, because it has only one parameter (except for the intercept  $s$ , which only depends on the initial weight  $Y_0$ ).

The shape of a complete growth curve of a (higher) plant, added to the preceding considerations, leads to an appreciation of the presuppositions underlying the logistic function.

These presuppositions are:

1. Rate of growth in any time interval is directly proportional to the actual weight  $Y_t$ .
2. Final weight approaches a level of saturation  $N$ .
3. Rate of growth in any time interval is directly proportional to the "tension"  $(N - Y_t)$ .

This can be expressed in the following differential equation:

$$\frac{dY}{dt} = aY_t(N - Y_t) \quad (a > 0) \quad (4)$$

After integration:

$$Y_t = \frac{N}{1 + be^{-ct}} \quad , \text{ where } c = aN \quad (5)$$

The logistic curve is asymptotic and symmetric. If  $Y_s$  is the point of inflexion:

$$Y_s = N/2 \quad (6)$$

$$t_s = \ln(b)/c \quad (7)$$

Although the set of presuppositions may be called -- from a biological point of view -- an excessive simplification, the logistic function seems to be an adequate model for the growth of (higher) plants under stable conditions.

Several methods are available for the estimation of the parameters of the logistic curve,  $c$ ,  $b$  and  $N$ , e.g. TINTNER (1960) and ERKELENS (1968). In practice these methods frequently give non-optimal estimations for  $b$  and  $N$ . The method proposed here is closely related to that described by Erkelens.

The first differences between the reciprokes of  $Y_t$  are logarithmically and linearly dependant to  $t$  (if time intervals are constant, e.g. a day, two days, a week):

$$\ln\left(\frac{1}{Y_t} - \frac{1}{Y_{t+1}}\right) = -ct + \ln(b/N) + \ln(1 - e^{-c}) \quad (8)$$

From this equation the growth coefficient  $c$  can be estimated by means of a regression analysis. Empirically it became evident that well-fitting estimations for  $b$  and  $N$  (final weight) in short time series are obtained by solving both

constants from the  $n$  available equations (5) two by two. So solve  $b$  and  $N$  from  $Y_1$  and  $Y_2$ , then  $Y_1$  and  $Y_3 \dots$  finally  $Y_{n-1}$  and  $Y_n$ . This yields  $\frac{1}{2}n(n-1)$  solutions. For each one of the sets of parameters the residual sum of squares can be computed by:

$$SSQ = \sum_{i=1}^n (Y_i - Y_i^*)^2 \quad (9)$$

( $Y_i^*$  is the calculated weight).

The lowest SSQ indicates the best-fitting logistic equation.

### 3. APPLICATION

As a demonstration the method has been applied to the growth of roots and entire plants of two annual species: *Galinsoga ciliata* (Raf.) Blake and *Galinsoga parviflora* Cav. Both species are neophytic annual weeds occurring in both pure and mixed populations, mostly under arable conditions. In this experiment plants were grown in a heated green-house in a potting soil/sand mixture. For data and results of computations see the *table*.

Table. Growth analysis of two species. Dry weight/grams ( $Y_t$ ) is the mean of a sample of 8, 9 or 10 plants,  $t$  weeks after germination.  $Y_t^*$  is weight calculated with best-fitting logistic equation with parameters  $c$ ,  $b$  and  $N$ .  $N'$  is  $N$ , calculated with Erkelens' method.

t	<i>G. ciliata</i> roots		<i>G. ciliata</i> plants		<i>G. parviflora</i> roots		<i>G. parviflora</i> plants	
	$Y_t$	$Y_t^*$	$Y_t$	$Y_t^*$	$Y_t$	$Y_t^*$	$Y_t$	$Y_t^*$
1	.03	.03	.11	.10	.01	.01	.03	.03
2	.09	.09	.26	.20	.03	.03	.09	.09
3	.19	.21	.46	.41	.06	.07	.27	.27
4	.40	.37	.90	.81	.14	.17	.86	.81
5	.51	.50	1.53	1.53	.38	.35	2.19	2.10
6	.55	.57	2.69	2.68	.58	.58	4.18	4.39
7	.60	.60	4.10	4.19	.78	.77	6.56	6.36
8	.61	.61	5.81	5.81	.88	.88	7.48	7.48
	c = 1.0845		c = 0.7292		c = 0.9994		c = 1.1640	
log.	b = 51.13		b = 185.44		b = 255.61		b = 966.49	
eq.	N = 0.6161		N = 8.9347		N = 0.9506		N = 8.2049	
	N' = 0.6854		N' = 12.0772		N' = 0.8243		N' = 5.1666	

Values of growth parameters show that roots of *G. ciliata* grow faster than those of *G. parviflora* ( $c = 1.0845$  resp.  $0.9994$ ), although the latter attain a larger dry weight ( $N = 0.6161$  g resp.  $0.9506$  g). For the entire plants the reverse seems to be true ( $c = 0.7292$  resp.  $1.1640$ ,  $N = 8.9347$  g resp.  $8.2049$  g). This may be an indication how selection in this genus has generated. Although

the final weight of *G. parviflora* is less, the higher rate of growth corresponds better to the environment of fast-growing crops. (Growth form – *G. ciliata* plants are lower and broader – may be of equal importance.)

A final remark: the *table* shows that values for N (and b), computed with Erkelens' method, are less adequate.

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