On the Logarithmic Frequency Curve and its Biological Importance

by

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When a study in biological statistics has advanced so far, that its results are expressed in the form of a frequency table or a variation polygon, we must by no means think that we have attained our end. It is desirable that these results should give us a deeper insight into the biology of our material.

In this connection I will not here treat of the coefficients expressing the degree of variability, but only of the conclusions to be drawn from the way in which the frequencies are distributed.

The fact that the broken line of a polygon does not satisfy our mind, and that we feel intuitively inclined to smoothen it into a curve, is justified by the following two considerations. First, there are irregularities due to the restricted number of individuals; with a similar but more numerous material these would gradually disappear. Secondly, the necessity of dividing the line of abscissae into a convenient number of intervals, gives to the line connecting the summits of the ordinates a broken character that is not inherent in the measured quality. In general we may suppose that the ideal distribution of frequencies would show a fluent line.

Now we may assume the existence of an analytical curve which would completely coincide with the ideal distribution just mentioned. As the equation of this curve would yield a surer basis for further inferences, it will be of great importance to obtain it.

Until now, a methodical inquiry into the type of this curve has seemed impossible. In practice every investigator, having a predilection for a definite type, tries to make a curve of that class agree as much as possible with the data. A perfect harmony between the analytical curve and the observations is never to be expected because of the irregularities in the latter. We can even imagine the case of two wholly different types both fitting equally well. An absolute criterion, which would enable us to decide which is the true type, is wanting. All we can do is to aim at a great probability that we are dealing with the true curve.

The reliability of our conclusions increases in proportion as the material is selected more carefully. This should be homogeneous, and also numerous: we need a far greater number than 300, which is generally considered sufficient for a variability coefficient.

It will be evident that the more a curve agrees with the data, the greater the probability of its being the true one. Investigators with decided mathematical bent, such as Pearson and his followers, are apt to regard the agreement as the only criterion. To me it would seem also of importance, if we could interpret our curve biologically, or in other words, if we could imagine causes which may have led to this special type of distribution.

This, then, is actually the case with Kapteyn's type of Skew Frequency Curves, for which I refer to his admirable mathematical treatment of the relations between causes and their effects [1]). I will cite only some of the

[1]) J. C. Kapteyn, Skew Frequency Curves in Biology and Statistics. Groningen, Noordhoff. 1903.

most essential points of his method, without some knowledge of which it would be impossible to understand clearly what I am going to say about the Logarithmic Curve, which will be treated as a special case of these Skew Curves.

Every individual has grown up under the influence of a complex of „causes", among which every amount of food, water, light, is to be reckoned. This complex has for each individual its own value; we will indicate it by the variable quantity z.

The distribution of these z's is supposed to follow a normal curve. If, notwithstanding this, the distribution of the measures x attained by the individuals, is found skew and not normal, this must be due to the special manner in which the organisms have undergone the influence of these causes, or reacted upon them.

When there is no hereditary diversity among the individuals, the correlation between causes and measures will be complete. If the ones vary independently, the others may be said to vary dependently. Let us express their connection by the equation

$$z = F(x).$$

Then, with an increase dz of the z, corresponds an increase dx of the x, so that

$$dz = F'(x)dx, \text{ or}$$

$$dx = \frac{1}{F'(x)}dz.$$

Here we see expressed the manner of reacting on the causes; the factor $\frac{1}{F'(x)}$ means that in general an increase of x during the period of growth depends on the value already attained.

K a p t e y n's method now consists in an endeavour to find a *normal* curve of z's related to the empirical skew curve of x's, sot hat equal frequencies exist between every two z's in the one, and their corresponding x's in the other, while the abscissae are values not of x itself, but of $z = F(x)$; or, in other words, a normal curve obtained by substituting for every x the corresponding value of a $z = F(x)$. If we succeed, the form of $F(x)$ will be known, and consequently also the „reaction factor" $\frac{1}{F'(x)}$, important for our knowledge of the physiology of growth.

So far no special supposition was made as to the type of $F(x)$. Kapteyn now introduces a type by which a considerable number of observed curves were found to be satisfactorily represented; namely

$$F(x) = (x + k)^q.$$

It implies, that the effect of normally distributed causes has been proportional to

$$\frac{1}{F'(x)} = \frac{1}{q} \cdot (x + k)^{1-q},$$

or simply to $(x + k)^{1-q}$, q being a constant.

The equation of the normal curve, being in its general form

$$y = \frac{h}{\sqrt{\pi}} \cdot e^{-h^2(z - M)^2}$$

now becomes

$$y = \frac{h}{\sqrt{\pi}} \cdot e^{-h^2[(x + k)^q - M]^2}$$

and that of the theoretical skew curve derived from it

$$y = \pm \frac{hq}{\sqrt{\pi}} \cdot (x + k)^{q-1} \cdot e^{-h^2[(x + k)^q - M]^2}.$$

To obtain the four parameters M, h, k, q, we choose on the line of abscissae four equidistant points $x_1 \ldots x_4$. Let Θ be the probability integral from the starting point of the curve to an x in general, then we get four equations by putting the Θ's of the theoretical curve in the points $x_1 \ldots x_4$ equal to those of the empirical one. If the solution succeeds, we may speak of a warranted fourfold coincidence between the two curves. Strictly speaking, there is equal frequency in *five* intervals, including also the interval past x_4.

Finally, by the aid of the constants, a theoretical value of Θ is to be computed for each x, and to be compared with the data.

Kapteyn's method is open to criticism, because of the arbitrary choice of the four x's. By taking other points, we shall obtain other constants. I have given some instances of this fact on p. 26—27 of my dissertation [1]. But, although mathematicians will assert that we can imagine better-fitting curves than Kapteyn's, still they cannot but consider it greatly to his credit that his method alone throws some light on the causes of variation, by very simple and yet correct reasoning [2].

If an empirical curve is normal itself, we have to do with the case $F(x) = x$, and $\frac{1}{F'(x)} = 1$. This implies that an increase of x has been independent of x.

Among the cases in which an increase does depend on x, the most simple one is a *direct proportionality* to the measure already attained, and it is this which leads to a *logarithmic* distribution. The condition

$$\frac{1}{F'(x)} = x$$

is satisfied by

$$F(x) = \log . x .$$

Consequently the curve generated under these circumstances is a skew curve, from which a normal one is obtained when we substitute every x by its logarithm. Hence the name.

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[1]) Statistische onderzoekingen bij *Senecio vulgaris* L. Groningen 1914.

[2]) A. O. Holwerda, Frequentiecurven. Diss. Utrecht 1913. p. 167—168.

The logarithmic curve has a historical interest, being the first skew frequency curve ever studied, namely by D. Mc. Alister [1]), on F. Galton's instigation [2]). (Originally the name of „Curve of Facility" was given to it.) Kapteyn supposes that this type „probably represents one of the most important classes occurring in nature" [3]). Even Pearson, in spite of his antagonism against Kapteyn's method in general, has no theoretical objections to this special curve, and discards it on account only of the unfavourable results of a series of testing-proofs [4]).

It is curious that the *physiological* considerations which led Galton to his discussion of the „Law of the Geometric Mean" should have impressed Pearson so deeply, that he also chose his test-objects from this domain and not from that of *variability*. Mc. Alister's treatment of the logarithmic curve is purely mathematical. Nobody seems to have tried earnestly its use in variation statistics. So there remained a gap to be filled up. Feeling a favourable disposition towards this curve because of the simplicity of its basis, I have been looking for instances.

Which are the characteristics that induce us to suppose a logarithmic distribution?

The skewness must be *positive*, that is, the ascent must be steeper than the descent.

The Median (*Med*) must be not the *arithmetic* but the *geometric* mean of the values q_1 and q_3 (25th and 75th

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[1]) The Law of the Geometric Mean. Proc. Roy. Soc. XXIX. 1879. p. 367.

[2]) Geometric Mean in Vital and Social Statistics. Proc. Roy. Soc. XXIX 1879. p. 365.

[3]) Skew Frequency Curves p. 22.

[4]) K. Pearson. „Das Fehlergesetz und seine Verallgemeinerungen durch Fechner und Pearson". A Rejoinder. Biometrika IV. 1905. p. 193—194.

Centesimal Grades, quartile-limits). Consequently, the two Quartiles must be unequal, $Q_2 > Q_1$; their ratio $\frac{Q_2}{Q}$ must be the same as $\frac{\text{Med}}{q_1}$ or $\frac{q_3}{\text{Med}}$.

Med must also be the geometric mean of every pair of Grades equidistant from 50 $^0/_0$. This will be understood from what follows. Two points on the line of abscissae of the normal *z*-curve, equidistant from its Mean (M), will include equal frequencies on either side of M. These two values, however, are the logarithms of the corresponding x's, and $M = log.Med$; thus in the skew curve the two x's including equal frequencies on either side of *Med*, will bear equal *ratios* to *Med*.

Here we see the importance of the Median, which is, in fact, the Geometric Mean of the whole series. Neither the Arithmetic Mean (lying to the right of it) nor the Mode (to the left) are of importance in the logarithmic curve.

The indications just mentioned were present in two frequency series from the material which I had collected for my dissertation, namely 300 plants of *Senecio vulgaris* L. For particulars about my material I refer to this publication. Let it suffice here to say that although in these wildgrowing plants no strict homogeneity was to be expected, I found in none of the studied characters a marked heterogeneity. I am quite sure that all the individuals belonged to one and the same subspecies.

For most of the characters I had taken only one measure per plant. These series were not numerous enough for analysis, but only for the computation of coefficients for variability and skewness. Fortunately, a few series were represented by more measures, and among these were the two instances which I am going to describe.

1st. *Top Cells of Pappus.*

The pappus hairs are composed of a small number of cell-rows, generally 4. As a rule, two of these reach the top. I had collected 275 capitula from as many plants, and on each of these I measured the length (see Fig. 1, *a—b*) of 10 top cells at random, avoiding any preference. There exists a wide variability even among the pappus of a single flower, hardly less than the total variability of these cells. So my series of 2750 lengths may be said to represent a typical scheme of distribution for this kind of cells. The curve was sufficiently fluent when I had divided the range of variation into 25 intervals. See Table 2.

I had first applied the general solution according to Kapteyn's method, and obtained an analytical curve which fitted properly. Its constants were $M = 3.430$; $h = 0.581$; $k = -9\frac{1}{3}$; $q = +0.7$.

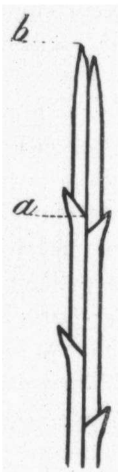

Fig. 1.

Wishing now to try a logarithmic curve, which represents the special case $k = 0$, $q = 0$, I found on p. 34 and 35 of the „Skew Frequency Curves" the formulae for the less special case $q = 0$ but k not 0. If a priori q is supposed to be 0, the equation contains only 3 parameters, for the solution of which only 3 values of x with their Θ's are required. The warranted coincidence is now but threefold; if there is found a more complete harmony, it is a check on the correctness of the supposition.

Similarly, if also k is supposed to be 0, there remain no more than *two* constants to be solved, by means of only two values of x and Θ. No more than twofold coincidence is warranted; but nobody will apply this solution unless a logarithmic distribution is suspected, and here, too, the correctness of the supposition will be proved by a more complete harmony.

The formulae for this case are very simple. The constant λ, being in general $\frac{x_1 + k}{i}$, now becomes $\frac{x_1}{i}$,

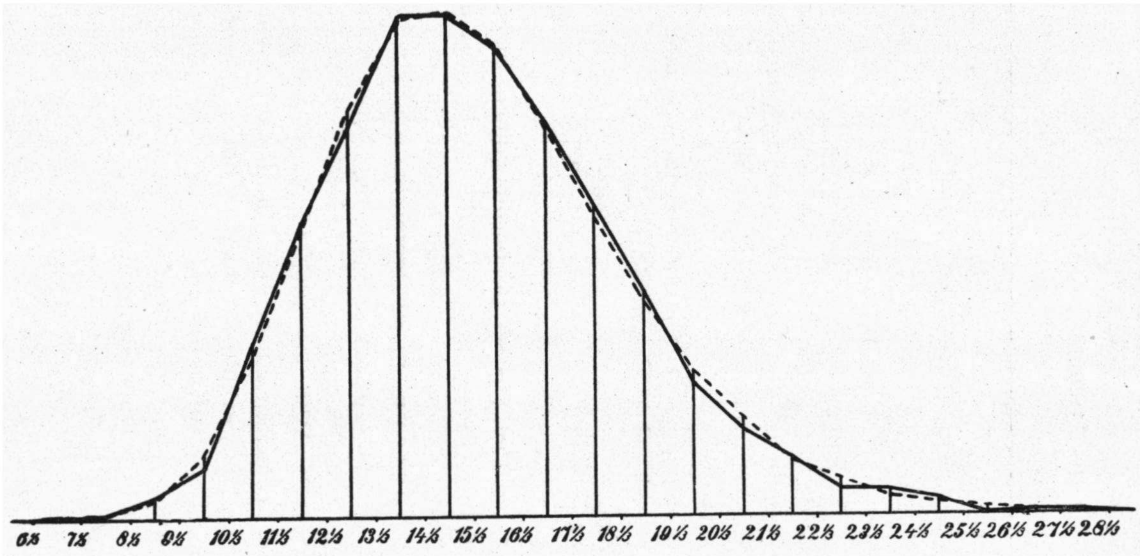

Fig. 2. Frequency-Curve of 2750 top cells of Pappus.

——— Empirical.

.... Logarithmic.

Scale of Ordinates: $1\,^0/_0 = 0.5$ cm.

Area of Curve: $^{100}/_3$ cm^2.

so that

$$x_1 = \lambda i, \qquad x_2 = (\lambda + 1)i.$$

λ is known from the beginning.

After looking up in Kapteyn's table 1b the two values R_1 and R_2 corresponding to the two Θ's we find M and h thus:

$$h = \frac{R_2 - R_1}{\log.(\lambda + 1) - \log.\lambda} = \frac{R_2 - R_1}{\log.\frac{x_2}{x_1}}.$$

$$M = -\frac{R_1}{h} + \log.\lambda + \log.i = -\frac{R_1}{h} + \log.x_1.$$

Finally, the equation of the analytical curve itself becomes

$$y = \frac{h}{\sqrt{\pi}} \cdot \frac{\text{mod}}{x} \cdot e^{-h^2(\log.x - M)^2}$$

For my series of pappus cells I obtained the constants

$$M = 1.1802; \quad h = 8.47;$$

and the equation

$$y = \frac{8.47}{\sqrt{\pi}} \cdot \frac{\text{mod}}{x} \cdot e^{-8.47^2(\log.x - 1.1802)^2}.$$

It will be noted that this set of constants differs widely from that obtained by the general solution. In fact, in the case $q = 0$ the parameters M and h have an altered meaning. (See Kapteyn, p. 20, footnote).

My Tables 1 and 2 give the comparison between the observations and the two solutions, General (Sol. 1) and Logarithmic (Sol. 2). With the latter, I had expected more than the twofold coincidence promised. But I had certainly not expected a harmony even *better* than that obtained by the general solution, especially at the ends of the curve. There are somewhat smaller differences, and *more points of contact.*

I conclude almost with certainty, that, in the case of my pappus cells, the logarithmic curve is the true one, so that the growth of these cells has been governed by this law: Increase directly proportional to the length already acquired.

2nd Instance. *Short bracts of the involučre.*

On the outside of the single row of long bracts there stand a rumber of much shorter ones, oval shaped, pointed, with a black-coloured top. The exact spot where the stalk of the capitulum begins to broaden into the receptacle, is not to be marked; ñeither is there a sharp limit between the bracts properly so called and the one or two leaflets often produced lower on the stalk. So I was

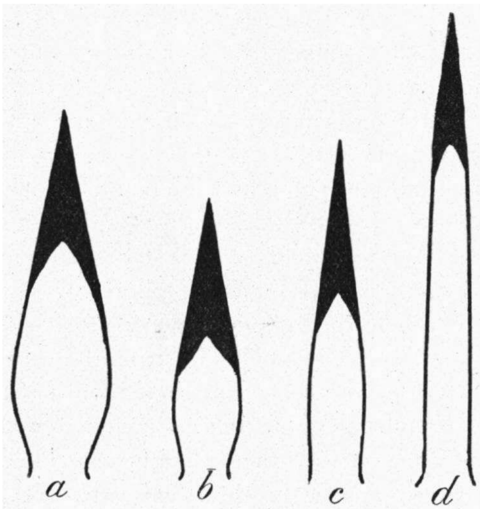

Fig. 3. Types of Bracts.
a. Lower transitional leaflet.
b. Typical bract.
c, d. Higher transitional leaflets.

obliged to include in my material of short bracts all the leaflets above the last branching. Likewise there exist, however rare, leaflets intermediate between short and long bracts. They approach the linear shape of the latter; but as long as they were placed below the edge on which the long bracts stand, I had to count them also among the short type.

On 268 capitula from as many plants I measured the length of all these short bracts, a total of 3685. Their empirical curve showed unmistakable positive skewness. So I tried a logarithmic curve obtained with two values of x near q_1 and q_3. ($x_1 = 13$, $x_2 = 17$). But this proved decidedly not to be the true one. It remained below the empirical curve in the terminal parts and above it in the middle: in short, it ran too steeply. The cause was obvious: the empirical curve had a long „tail", due to the transitional leaflets being no true bracts but of greater length than these.

I now wanted to make out if perhaps a logarithmic curve would agree with the parts where no influence of these plus-variates is yet felt. Kapteyn's method does not provide for this case, as it deals only with homogeneous material. So I had to look for a solution myself, and I succeeded by reasoning as follows.

Let there be, mixed up with every 100 typical individuals, a number of a which differ from the type: together $100 + a$. If a curve were constructed only of the typical individuals on such a scale that it included with the line of abscissae an area 100, then it might coincide completely with a theoretical curve of this same area 100. By now adding the nontypical individuals, the area of the empirical curve becomes $100 + a$; but its left part (where none of these plus-variates occur) continues to coincide with the theoretical curve, of which the area is only 100.

If we now choose two values x_1 and x_2 in the region where only typical individuals occur, and if Θ_1 and Θ_2 represent the percentage of observations in the *whole* material below these values, then of the *typical* individuals $\frac{100 + a}{100} \times \Theta_1$ lie below x_1, and $\frac{100 + a}{100} \times \Theta_2$ below x_2. These reduced Θ's are to be the values of the probability integral for x_1 and x_2 in the theoretical curve that represents

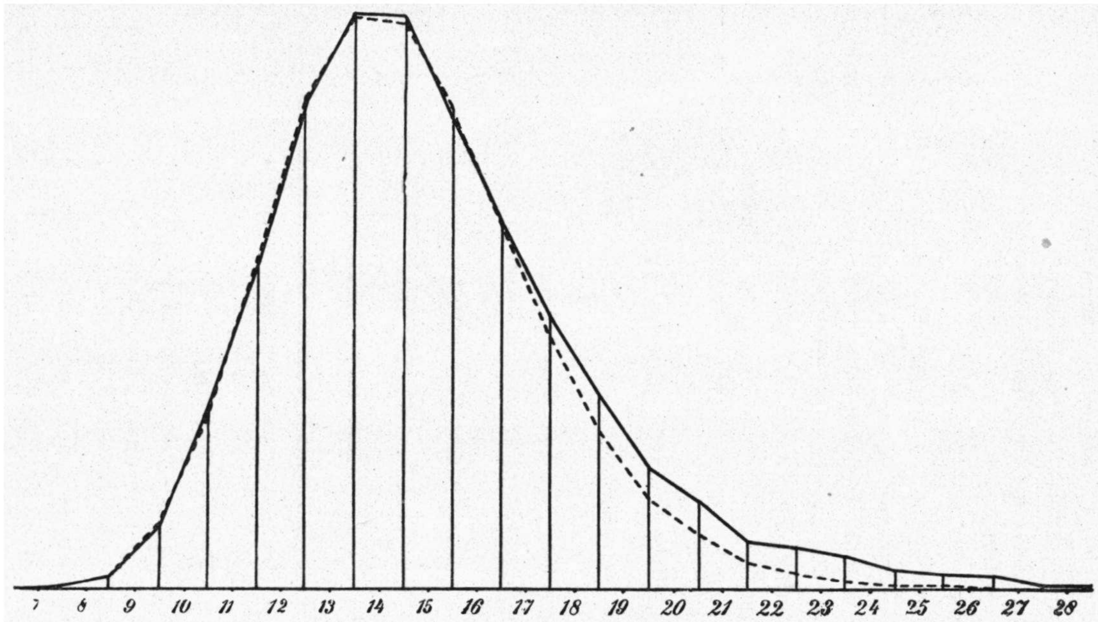

Fig. 4. Frequency-curve of 3685 Bracts. ———— Empirical. .... Logarithmic.
Scale of Ordinates: 1 % Emp. = 0.535 cm. 1 % Log. = 0.500 cm.
Area of Curves: Empirical $^{107}/_{8}$ cm². Logarithmic $^{100}/_{8}$ cm².

only the type. Consequently the *reduced* Θ's are to be used for the solution of the constants.

Unfortunately the number a is not known; neither is the factor $\frac{100+a}{100}$ by which the empirical Θ's are to be multiplied. But as soon as we assume a definite value for a, the solution can be carried through. Different values of a will of course yield different curves; the higher the value, the steeper the curve. We may now fix the condition that ihe empirical and theoretical curve shall coincide at a third point, whereby a becomes a sort of parameter to be determined.

I first tried the value $a = 10$, thus $\frac{100+a}{100} = 1{,}1$.

This gave:

Values of x used	$x_1 =$	13	$x_2 =$	17
Empirical Θ's	$\Theta_1 =$	0.274	$\Theta_2 =$	0.779
Reduced Θ's (1.1 × Θ)		0.302		0,857
Values of R from Table 1b	$R_1 =$	— 0.367	$R_2 =$	+ 0.755

Hence: $h = \frac{0.367 + 0.755}{\log.\frac{17}{13}} = 9.63$

$$M = \frac{0.367}{9.63} + \log.\ 13 = 1.15205.$$

With these constants I computed the theoretical value of Θ for $x = 15$, a point equidistant between x_1 and x_2.

$$R = h(\log. x - M) = +\ 0.232$$

$$\text{Reduced } \Theta = 0{,}628;\ \Theta = 0.571.$$

Empirical Θ = 0.566. Difference Emp. — Theor. = — 0.005.

The original logarithmic curve had given at this point a difference + 0.014 between Empirical and Theoretical Θ. I expected that the value for a obtained by interpolation, namely a = about 7, would yield the desired harmony. I got:

$$h = 9.18;\quad M = 1.15575.$$

As seen in Table 3 and 4, the series of computed Θ's, and also the series of frequencies, now agree admirably with the empirical data (reduced numbers) as far as the point $x = 17$. From here the theoretical Θ's approach the final value 1.000; whereas the empirical Θ's increase more rapidly to the final value 1.070, and diverge more and more from the former, since this is the region where we meet the non-typical leaflets. Their number in each interval is expressed by the difference between the ordinates of the frequency curves.

These results justify the following conclusion:

About $\frac{100}{107} = 93\,\%$ of the bracts in question belonged to a type governed fully by the Law of the Geometric Mean. The remaining $7\,\%$ (in this case about 241) were non-typical, transitional leaflets, some inserted lower and the rest higher, but all of greater length.

I have also measured the breadth of these same bracts, and I should like to have treated this series in an analogous manner. But in this respect the lower transitional leaflets were extreme variates on the plus side, being shaped like normal bracts but more robust; on the contrary the more linear-shaped higher leaflets were extreme variates on the minus side. I have ascertained the fact that, in comparison with a logarithmic curve, the empirical distribution shows „a tail" on either side. Although I suppose that the middle part would follow a logarithmic distribution, I have not tried to find the theoretical curve, as it would never afford a *striking* instance.

And this alone I intended to show: the existence of striking cases where it is almost certain that organs have been governed by the Law of the Geometric Mean.

It is my personal conviction, that Logarithmic distribution represents a more frequent case in nature than even the Normal curve. Among 145 frequency series from my own material no less than 111 showed positive skewness. The numbers were mostly too small for a trustworthy analysis, as stated before; but I consider the predominance of positive skewness as a very strong indication of a more frequent occurrence of Logarithmic distribution than has hitherto been supposed.

Arnhem, May 1915.

Table I. Pappus-cells.

Integrals.

x Unit = 0.01056 mm.	Observed.		Computed.		Diff. Obs—Comp.	
	Indiv. below x	Θ	Sol. 1.	Sol. 2.	Sol. 1.	Sol. 2.
$6^1/_3$	1	0.000	0.000	0.000	0.000	0.000
$7^1/_3$	3	001	000	000	+ 1	+ 1
$8^1/_3$	5	002	000	001	+ 2	+ 1
$9^1/_3$	22	008	002	006	+ 6	+ 2
$10^1/_3$	61	022	023	023	— 1	— 1
$11^1/_3$	189	069	070	066	— 1	+ 3
$12^1/_3$	408	148	**148**	143	0	+ 5
$13^1/_3$	699	254	258	**254**	— 4	0
$14^1/_3$	1070	389	**388**	388	+ 1	+ 1
$15^1/_3$	1443	525	524	525	+ 1	0
$16^1/_3$	1793	652	**651**	653	+ 1	— 1
$17^1/_3$	2087	759	759	**759**	0	0
$18^1/_3$	2319	843	**843**	840	0	+ 3
$19^1/_3$	2487	904	903	898	+ 1	+ 6
$20^1/_3$	2585	940	943	937	— 3	+ 3
$21^1/_3$	2649	963	968	963	— 5	0
$22^1/_3$	2693	979	983	978	— 4	+ 1
$23^1/_3$	2712	986	992	988	— 6	— 2
$24^1/_3$	2732	993	996	993	— 3	0
$25^1/_3$	2743	997	998	996	— 1	+ 1
$26^1/_3$	2744	998	999	998	— 1	0
$27^1/_3$	2747	999	1.000	999	— 1	0
$28^1/_3$	2749	1.000	1.000	1.000	0	0
$29^1/_3$	2750	1.000	1.000	1.000	0	0

N.B. The numbers in bold-faced type were used for the solutions.

Table 2. Pappus-cells.

Frequencies.

x Unit = 0.01056 mm.	Observed.		Computed.		Diff. Obs—Comp.	
	Indiv.	‰	Sol. 1	Sol. 2	Sol. 1	Sol. 2
	1	0.000	0.000	0.000	0.000	0.000
$6^{1}/_{3}$						
	2	001	000	000	+ 1	+ 1
$7^{1}/_{3}$						
	2	001	000	001	+ 1	0
$8^{1}/_{3}$						
	17	006	002	005	+ 4	+ 1
$9^{1}/_{3}$						
	39	014	021	017	— 7	— 3
$10^{1}/_{3}$						
	128	047	047	043	0	+ 4
$11^{1}/_{3}$						
	219	079	078	077	+ 1	+ 2
$12^{1}/_{3}$						
	291	106	110	111	— 4	— 5
$13^{1}/_{3}$						
	371	135	130	134	+ 5	+ 1
$14^{1}/_{3}$						
	373	136	136	137	0	— 1
$15^{1}/_{3}$						
	350	127	127	128	0	— 1
$16^{1}/_{3}$						
	294	107	108	106	— 1	+ 1
$17^{1}/_{3}$						
	232	084	084	081	0	+ 3
$18^{1}/_{3}$						
	168	061	060	058	+ 1	+ 3
$19^{1}/_{3}$						
	98	036	040	039	— 4	— 3
$20^{1}/_{3}$						
	64	023	025	026	— 2	— 3
$21^{1}/_{3}$						
	44	016	015	015	+ 1	+ 1
$22^{1}/_{3}$						
	19	007	008	010	— 1	— 3
$23^{1}/_{3}$						
	20	007	004	005	+ 3	+ 2
$24^{1}/_{3}$						
	11	004	002	003	+ 2	+ 1
$25^{1}/_{3}$						
	1	000	001	002	— 1	— 2
$26^{1}/_{3}$						
	3	001	001	001	0	0
$27^{1}/_{3}$						
	2	001	000	001	+ 1	0
$28^{1}/_{3}$						
	1	000	000	000	0	0
	2750	0.999	0.999	1.000		

Table 3. Bracts,

Integrals.

x Unit = 0.144 mm.	Observed.			Computed Θ	Difference Red—Comp.
	Indiv. below x	Θ	Reduced (1.07 Θ)		
6	1	0.000	0.000	0.000	0.000
7	1	000	000	000	0
8	4	001	001	001	0
9	15	004	004	004	0
10	74	020	021	022	− 1
11	247	067	072	069	+ 3
12	552	150	160	160	0
13	1011	274	294	294	0
14	1550	421	450	450	0
15	2087	566	606	604	+ 2
16	2528	686	734	735	− 1
17	2872	779	834	834	0
18	3127	849	908	902	+ 6
19	3309	898	961	945	+ 16
20	3423	929	994	970	+ 24
21	3506	951	1.018	985	+ 33
22	3549	963	031	992	+ 39
23	3587	973	042	996	+ 46
24	3618	982	051	998	+ 53
25	3638	987	056	999	+ 57
26	3652	991	060	1.000	+ 60
27	3661	993	063	000	+ 63
28	3665	995	064	000	+ 64
29	3667	995	065	000	+ 65
30	3669	996	066	000	+ 66
∞	3685	1.000	1.070	1.000	+ 0.070

Table 4. Bracts.

Frequencies.

x Unit = 0.144 mm.	Observed: Indiv.	Observed: y in ‰	Observed: Reduced (1.07 y)	Computed ‰	Difference Red—Comp	
6	1	0.000	0.000	0.000	0.000	
7	—	000	000	000	0	
8	3	001	001	001	0	
9	11	003	003	003	0	
10	59	016	017	018	— 1	
11	173	047	050	047	+ 3	0
12	305	083	089	091	— 2	
13	459	125	133	134	— 1	0
14	539	146	157	156	+ 1	
15	537	146	156	154	+ 2	
16	441	120	128	131	— 3	0
17	344	093	100	099	+ 1	
18	255	070	074	068	+ 6	
19	182	049	053	043	+ 10	
20	114	031	033	025	+ 8	
21	83	023	024	015	+ 9	
22	43	012	013	007	+ 6	
23	38	010	011	004	+ 7	
24	31	008	009	002	+ 7	0.072
25	20	005	006	001	+ 5	
26	14	004	004	001	+ 3	
27	9	002	003	000	+ 3	
28	4	001	001	000	+ 1	
29	2	001	001	000	+ 1	
30	2	001	001	000	+ 1	
	16	004	005	000	+ 5	
	3685	1.001	1.072	1.000		