## PROFESSOR MOLL'S METHOD OF DETERMINING THF AMOUNT OF SUNSHINE ON GREENHOUSES.

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Among the posthumous writings of Prof. Dr. J. W. Moll in the Groningen University Museum one rather comprehensive manuscript is to be found, dating from the year 1915, on the Groningen University Botanic Garden.

This manuscript which for several reasons has never been published, contains a paragraph on the determination of the hours during which the sun, when shining, may reach by its direct beams a given greenhouse or any particular object on any day of the year.

Of course direct observation on the spot, continued during a sufficient period, may yield the same result, but for many purposes a method giving a reliable survey at once may have great advantages over the protracted and tedious direct observation.

Botanists or plant growers wanting to erect green- or hothouses may especially profit from a comparison in this respect of the several available spots on their premises.

For the culture of tropical plants in a temperate climate the amount of sunlight during the winter months often may have a preponderant importance; for many other cultures the morning. and evening light in summer may be especially valuable, particularly when one is obliged to shade off the house in the middle of the day for fear of overheating or sunburning.

Considering that this paragraph of Moll's manuscript might be of interest for many botanists the present Director of the Groningen Garden, Prof. Dr. W. H. A ris z, requested me to publish this part separately, a task which I undertook the more readily as I had been in touch with the development of Prof. Moll's ideas on the topic from the very beginning, being his assistant in the year 1900, the time he first began to ponder about it.

The method devised by Moll consists in the construction of two diagrams, a general one for the sun's course under the latitude of the place, and another for every particular spot to be investi-
gated representing the silhouette of the sunlight impeding objects against the sky.
If these diagrams are plotted according to certain rules and if the silhouette diagram is drawn on transparent paper, a simple superposition of the two diagrams will reveal at once the hours of possible sunshine for the spot in question during the whole year.

The construction of these diagrams runs as follows. A circle with a radius of e.g. 10 cms represents the horizon around the spot in question, which is supposed to occupy the centre. The curves of the sun's course as well as the silhouette of the surrounding objects are represented as rabatted on the plane of the drawing outside the circle, in the following way.

For every sun's position and for every point of the silhouette the azimuth is determined and the altitude at which it is to be seen from the spot in question. A radius from the centre $C$ with the same azimuth is then produced outside the circle to a length equal to the tangent of the said altitude, the radius of the circle being unity.

We may describe this operation in other terms in the following way. With the circle as a base, a perpendicular cylinder is supposed to be erected on which the sun's course as well as the neighbouring objects are projected. This cylinder is then rabatted on the horizontal plane outside the circle and all curves on the cylinder are transformed into analogous curves in the horizontal plane, only somewhat deformed by the increased girth.

For the construction of the curves of the sun's course the following formulae ${ }^{1}$ ) were used:
$\sin \mathrm{h}=\sin \mathrm{L} \sin \delta+\cos \mathrm{L} \cos \delta \cos \mathrm{t}$
$\sin \mathrm{A}=\cos \delta \sin \mathrm{t}: \cos \mathrm{h}$
$\cos \mathrm{T}=-\tan \mathrm{L} \tan \delta$,
in which $\mathrm{h}=$ sun's altitude, $\mathrm{L}=$ latitude of place, $\delta=$ sun's apparent declination, to be found in any Nautical Almanac for every day of the year, $t=$ hour angle, $T=$ hour angle at sunrise or sunset, $\mathrm{A}=$ sun's azimuth.

The hour angle is $0^{\circ}$ at noon, $15^{\circ}$ at $\mathrm{r} 0^{\prime}$ clock p.m. and - $15^{\circ}$

[^0]at in a.m., all hours being true hours, not the usual mean time. These hours may differ from the mean time up to a quarter of an hour; their advantage however is that at noon true time the sun is actually placed in the meridian.

The sun's azimuth is $0^{\circ}$ at noon, and is negative a.m. and positive p.m.

In practice it will mostly do to calculate the sun's course for I2 days in the year, distributed as evenly as possible over the months and including the solstices: for the construction of a single curve for a particular day it will be sufficient to calculate the position for every full hour and moreover for sunrise and sunset; as the calculation for the hours a.m. and the corresponding hours p.m. is the same, the operation does not take up much time.

As an instance the calculation for 22 December for the latitude of Groningen may follow.
$\mathrm{L}=53^{\circ}{ }^{12^{\prime}} \mathrm{N} ; \sin \mathrm{L}=0.8008 ; \cos \mathrm{L}=0.5990 ; \tan \mathrm{L}=\mathrm{r} .337$. $\delta=-23^{\circ}{ }^{2} 6^{\prime} ; \sin \delta=-0.3977 ; \cos \delta=0.9175 ; \tan \delta=-0.4334$. $\sin \mathrm{L} \sin \delta=-0.3187$.
$\cos \mathrm{T}=-\mathrm{r} .337 \times-0.4334=0.578^{2}$ ) $; \mathrm{T}= \pm 54^{\circ} 4 \mathrm{r}^{\prime}=8.2 \mathrm{r}$ a.m. and 3.39 p.m.

| Hour | t | $\cos t$ | $\cos \delta \cos \mathrm{t}$ | $\cos \mathrm{L} \cos \delta \cos \mathrm{t}$ | $\sin h$ | h |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 12 | 0 | 1.000 | 0.9175 | 0.5496 | 0.2309 | $13^{\circ} 21^{\prime}$ |
| $1^{3}$ ) | $15^{\circ}$ | 0.9659 | 0.8863 | 0.5309 | 0.2122 | $12^{\circ} 16^{\prime}$ |
| 2 | $30^{\circ}$ | 0.8660 | 0.7946 | 0.4760 | 0.1573 | 9031 |
| 3 | $45^{\circ}$ | 0.7071 | 0.6847 | 0.3886 | 0.0699 | $40^{\circ} 0^{\prime}$ |
|  |  |  |  |  |  |  |
| Hour | t | $\sin t$ | $\boldsymbol{\operatorname { c o s }} \delta \boldsymbol{\operatorname { s i n }} \mathrm{t}$ | $\cosh$ | $\sin \mathrm{A}$ | A |
| $\left.1^{4}\right)$ | $15^{\circ}$ | 0.2588 | 0.2376 | 0.9771 | 0.243 | $1405^{\prime 6}$ |
| 2 | $30^{\circ}$ | 0.5000 | 0.4587 | 0.9875 | 0.4650.650 | 27040 |
| 3.39 | $45^{\circ}$ | 0.7071 | 0.6487 | 0.9976 |  | $40^{\circ} 30^{\prime \prime}$ |
| 3 | $54^{\circ} 41^{\prime}$ | 0.9175 | 0.7487 | 1.0000 | 0.7487 | $48^{\circ} 29^{\prime}$ |

${ }^{2}$ ) For $\delta=$ positive, cos T becomes negative and $\mathrm{T}>90^{\circ}$.
${ }^{3}$ ) For 11 a.m. $t$ being $-15^{\circ}, \cos t$ and all subsequent figures are the same as for 1 p.m.
${ }^{4}$ ) For 11 a.m. $t$ being $-15^{\circ}, \sin t, \cos \delta \sin t, \sin A$ and $A$, though retaining the same numerical values, all become negative.
${ }^{5}$ ) The value of A found here is either $14^{\circ} S^{\prime}$ or $165^{\circ} 55^{\prime}$ of which the first value obviously only meets the reality. When however there might be any doubt whether A should be $>$ or $<90^{\circ}$ the additional formula $\cos \mathrm{A}=$ $-\cos L \sin \delta+\sin L \cos \delta \cos t: \cos h$ may be used.

From this calculation and the analogous ones for the other chosen days the following seven tables are obtained in which all values have been somewhat rounded off.

Sun's altitude and azimuth for $L=\int 3^{\circ} 12^{\prime} \mathrm{N}$.

| December 22 |  |  |  | April 22 and August 22 |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Hour | h | A | $\tan \mathrm{h}$ | Hour | h | A | $\tan h$ |
| noon | $131 /{ }^{\circ}$ | $0^{\circ}$ | 0.237 | noon | 489,9 | $0^{\circ}$ | 1.141 |
| 11 and | 121/4 ${ }^{\circ}$ | $14^{\circ}$ | 0.217 | 11 and | $47^{\circ}$ | $2134^{\circ}$ | 1.075 |
| 10 . | $90^{\circ}$ | $273 / 4^{\circ}$ | 0.159 | 10 . | $421 /{ }^{\circ}$ | $411 / 2^{\circ}$ | 0.912 |
| 9" 3 | $4^{\circ}$ | $4012^{\circ}$ | 0.070 | 9 " | $351 \%{ }^{\circ}$ | 581/4 ${ }^{\circ}$ | 0.713 |
| 8.21 ". 3.39 | $0{ }^{\circ}$ | 481\% ${ }^{\circ}$ | 0.000 | 8 " | $2714^{\circ}$ | $7212^{\circ}$ | 0.517 |
| January 22 and November 21 |  |  |  | 7", 6 | $181 / 2^{\circ}$ $91 / 22^{\circ}$ | $8434^{\circ}$ $971 / 2^{\circ}$ | 0.335 0.168 |
| noon | 17 | 0 | 0.306 | 4.54", 7.06 | $0{ }^{\circ}$ | $1090^{\circ}$ 110 | $\begin{aligned} & 0.014 \\ & 0.000 \end{aligned}$ |
| 11 and | 153/40 | $143 / 4^{\circ}$ | 0.284 | May 23 and July 22 |  |  |  |
| 10 " 2 | $121 /{ }^{\circ}$ | $283 / 4^{\circ}$ | 0.222 |  |  |  |  |
| $9 \% \quad 3$ | $71 / 4{ }^{\circ}$ | $421^{\circ}$ | 0.129 |  |  |  |  |
|  | $0^{1 / 2}{ }^{\circ}$ | 5412 ${ }^{\circ}$ | 0.011 | noon | 571/4 ${ }^{\circ}$ | $0^{\circ}$ | 1.554 |
| 7.55 , 4.05 |  | $5512^{\circ}$ |  | 11 and | 551/4 ${ }^{\circ}$ | 25140 | 1.442 |
| February 21 and October 22 |  |  |  | 10 " 2 | 50 ${ }^{\circ}$ | 441120 ${ }^{\circ}{ }^{\circ}$ | 1.191 0.918 |
| noon | 261/2 ${ }^{\circ}$ | $0^{\circ}$ | 0.501 | $8 \% \quad 4$ | $34^{\circ}$ | $7812^{\circ}$ | 0.675 |
| 11 and 1 | 243/4 ${ }^{\circ}$ | $1614^{\circ}$ | 0.461 | $7 \% \quad 5$ | $25^{\circ}$ | 893/4 ${ }^{\circ}$ | 0.469 |
| 10 , 2 | $21^{\circ}$ | $31^{3 / 4}{ }^{\circ}$ | 0.385 | ${ }_{6}^{6} \times 6$ | 161/40 | $10212^{\circ}$ | 0.291 |
| $9 \% \quad 3$ | $15^{\circ}$ | 451/4 ${ }^{\circ}$ | - 268 | $\begin{array}{r}5 \\ 4.01 \\ \hline 1 . \\ \hline\end{array}$ | 73/4 ${ }^{\circ}$ |  | 0.135 0.000 |
| 8 " ${ }^{\prime \prime}$ | $8^{\circ}$ | $591 / 4^{\circ}$ | 0.147 | 4.01 . 7.59 | $0^{\circ}$ | $1253 / 4^{\circ}$ | 0.000 |
| 7 ${ }^{\text {" }}$ " 505 | $0^{\circ}$ | $7112^{\circ}$ | 0.001 | June 22 |  |  |  |
| 6.59 , 5.01 | $0^{\circ}$ | $713 / 4{ }^{\circ}$ | 0.000 |  |  |  |  |
| March 23 and Seplember 21 |  |  |  | noon11$10^{\text {and }}$0 | $\begin{aligned} & 601 / 4^{\circ}{ }^{\circ} 581 / 4^{\circ} \\ & 4934^{\circ} \end{aligned}$ | $0^{\circ}$ | 1.749 |
| noon |  | $0^{\circ}$ | 0.772 |  |  | 45120 | 1.307 |
| 11 and | $361 / 4^{\circ}$ | $183 / 4^{\circ}$ | 0.731 | 9." 3 | $45^{\circ}$ | $66^{3} / 4^{\circ}$ | 1.000 |
| 10 " | $32^{\circ}$ | $361 / 4^{\circ}$ | 0.626 | $8 \% \quad 4$ | $361 / 2^{\circ}$ | 8034 ${ }^{\circ}$ | 0.737 |
| $9 \% \quad 3$ | $253 / 4^{\circ}$ | $51^{3} 4^{\circ}$ | 0.481 | $7 \% 5$ | 270 | 9534\% ${ }^{\circ}$ | 0.519 |
| 8 \% ${ }^{\prime \prime}$ - 4 | $181 / 4^{\circ}$ | 65\% ${ }^{\circ}$ | 0.328 | $6 \cdots \quad 6$ | 181/20 | $1043{ }^{\circ}{ }^{\circ}$ | 0.334 |
| $7 \% \quad 5$ | 98/4* | $781 \%^{\circ}$ | 0.170 |  | 1012 ${ }^{\circ}$ | 1158/4 ${ }^{\circ}$ | 0.185 |
| 6 \% ${ }^{\prime}$. 6 | 3/4 ${ }^{\circ}$ | $90^{\circ}$ | 0.012 | $4 \% r 8$ | $21 / 2^{\circ}$ | 1271/4 ${ }^{\circ}$ | 0.044 |
| 5.55 , 6.05 | 00 | 903/4 ${ }^{\circ}$ | 0.600 | 3.38 " 8.22 | $0^{\circ}$ | $13112^{\circ}$ | 0.000 |

From these tables the curves for the sun's course may easily be plotted (see Fig. I).

The construction of a silhouette of the sunlight impeding objects for any particular spot requires the determination of the altitudes at which the objects are seen from the spot and moreover the deter-


Diagram of the suns course for Groningen (lat. $53^{\circ} 12^{\prime} \mathrm{N}$.), with indication of the principal sunlight impeding objects around the hothouse №3 of the Groningen University Botanical Garden, and their silhouette against the sky
mination of the azimuth of the objects. This may be done by means of a sextant and a compass.

In our figure such a silhouette has been inserted as a shaded area, a silhouette which had been plotted for Prof. Moll in 191s by his assistant Dr. B. Havinga and which gives the conditions for that one of the Groningen hothouses which as shown by the gardener's experience had the best situation.

Yet our figure demonstrates clearly that from Nov. 21 until Jan. 22 only the smaller part of the available sunlight might reach the house, on Dec. 22 from about 10 a.m. to not quite in and from 12.10 to 12.30 .

We may further see that the loss of light is partly due to the presence of the Botanical Laboratory, but for the greater part to some trees in the garden (L. P. a Lombardy poplar, As two ashtrees, Oa an oak, B a birch and W. H. a white horsechestnut), and we may easily estimate the gain in light the removal of any of these trees would yield.

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[^0]:    ${ }^{1}$ ) Originally Prof. Moll determined these curves without any formulae or calculation by the aid of a celestial globe; the sun's position for a certain day being marked on the globe, the principal points of the curves were read off at once by means of two meridians, one a fixed meridian of the globe and one a moveable meridian, fixed in the zenith of the place.

    As this method is less accessible for most botanists the use of the formulae is to be preferred, the more as the results of the observation on the globe at not nearly so accurate, so that for the construction of smooth curves the figures have to be manipulated with a good deal of freeness.

